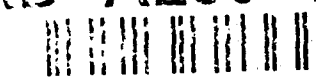

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The DOE CHAMMP programs stated goal is to develop, verify, and apply a new generation of climate model within a coordinated framework. The JASON review of the CHAMMP program took place during early July 1991.

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Contents

1 INTRODUCTION

The U.S. global change research program embraces three important classes of activities. One is the long-term monitoring of climatic variables to detect and measure global change. A second class comprises detailed short-term studies of the processes that govern the climate system. The third activity involves the construction of climate models (e.g., ocean and atmospheric general circulation models – GCMs) incorporating the results of process studies to improve the ability to predict and eventually to respond to global change. The CHAMMP program clearly falls within the third class of activities (modeling) and complements DOE's Atmospheric Radiation Measurement Program (ARM) of process studies in the atmosphere, and NASA's Earth Observing System (EOS) program, aimed primarily at monitoring.

CHAMMP's stated goal is "To develop, verify, and apply a new generation of climate model within a coordinated framework that:

1. incorporates the best available scientific and numerical approaches to represent physical, biogeochemical, and ecological processes;

2. fully utilizes the hardware and software capabilities of new computer architectures;

3. probes the limits of climate predictability;

4. can be used to address the challenging problem of understanding the greenhouse climate issue through the ability of models to simulate time-

dependent climatic changes over extended times with regional resolution."

The JASON review of the CHAMMP program took place during early July 1991. This review continues the role that JASON has played in advising program management and the CHAMMP participants on the scientific and programmatic issues. Our report from last year can be found in JSR-90-306. Input to the present review came in the form of briefings on various scientific aspects of CHAMMP by scientists involved in the program and by other members of the climate modeling community on July 13, 1991. We also heard a full day of presentations on the accomplishments, status, and prospects for the program given by the CHAMMP management team on July 12. Additional written material was provided to us by the CHAMMP management. Several JASONs also have familiarity with the program through informal contacts with CHAMMP management and scientists over the past year.

Following a brief discussion of program accomplishments (Section 2), we examine issues raised in highly parallel computing (Section 3). We next take up the questions associated with predictability (Section 4), and present our assessment of the program (Section 5). Possible future directions of research are discussed in Section 6. The links between CHAMMP and other elements of the Department of Energy's global change programs are considered in Section 7. The report also contains four technical appendices, which are included to illustrate certain programmatic issues discussed in the main body of the report. Appendix A presents one attempt to narrow the definition

of what is meant by predictability of climate. The questions of coherent structures and spatial statistics are discussed in Appendix B. A fluctuation-correlation relationship is presented in Appendix C. Appendix D illustrates the potential contribution from mathematics by providing a mathematical argument on the nature of extreme events in dynamical systems that can be represented by an attractor.

2 PROGRAM ACCOMPLISHMENTS

The first step in the CHAMMP program strategy is to adapt the current generation of GCMs to high-performance parallel computers. At the time of our first review of CHAMMP, the program was just beginning. During the past year, there have been a number of solid scientific accomplishments that bode well for the future of the program.

A major accomplishment is the implementation of the NCAR Community Climate Model (CCM-2) on the Intel family of parallel supercomputers. This is a full spectral atmospheric GCM [Legendre transform in the latitude, Fourier transform in the longitude, and finite difference in the vertical coordinate] and so is not naturally adapted to a multiprocessor architecture. Nevertheless, it is running on both a 128 node Gamma and a 540 node Delta computer and validation is underway. It is estimated that a 1-year simulation at a resolution of T42 will take 2 hours on the Delta, while the same calculation at T213 will take 50 hours.

A second major accomplishment is the implementation of the Semtner-Chervin Ocean GCM on the single instruction/multiple data (SIMD) CM-2 computer. This is a grid-point model with an eddy resolving (0.5°) horizontal resolution, 20 vertical levels, and a realistic bottom topography. Performance comparable to that of an 8-processor Cray YMP has been achieved, due in part, to a new method for solving the surface pressure equation. Further improvements are expected. This same model has also been implemented on

the multiple instruction/multiple data (MIMD) BBN parallel architecture, and there has been an extensive study of the efficiency of grid-point methods for a shallow water problem on that architecture.

We also note that, within the last year, the CHAMMP program has awarded the equivalent of 13,000 Cray-1 hours, has solicited, reviewed, and awarded a number of Science Team proposals, and has completed Program, Management, and Implementation plans.

3 ISSUES IN HIGHLY PARALLEL COMPUTATIONS

There has been much discussion of a variety of highly parallel computing hardware architectures in recent years. The biggest difference has been between the SIMD architecture of, for example, the Connection Machine 2 (CM-2) and the MIMD architectures of machines produced, for example, by Intel and BBN. SIMD is clearly more restrictive than MIMD, but it is particularly easy to use for regular mesh calculations, especially through the data parallel approach embodied in the new FORTRAN-90 compiler. The greater flexibility of MIMD provides more freedom of choice in programming models, but at the price of complexity. It has long been realized that a SIMD machine can be easily simulated on a MIMD machine, but not the reverse.

The additional hardware cost of MIMD over SIMD capability is negligible, and thus it is not surprising that, in the next few years, the evolving hardware will be MIMD, though with compilers it will provide the ease of SIMD data parallel use. The issue then will be to decide the most appropriate programming model for a given application. The two main choices in the MIMD world are to treat all memory as though it were shared by each processor node, or to approximately more closely the actual hardware by treating the memory as distributed among the individual nodes, with explicit message passing to transfer data between the local memories of the nodes.

Atmosphere and ocean models based on grid-point methods seem particularly well suited to a two-dimensional domain decomposition message-passing programming model on MIMD machines. For this model, the whole code is subdivided horizontally into regions, each of which is assigned to a single node. For commonly used explicit differencing schemes, it is then only necessary to exchange border data by passing messages between contiguous regions. The relative cost of data transfer is diminished in this way by a perimeter-to-area ratio. This cost must be minimized for efficient use of parallel hardware.

There are two advantages to such a distributed memory message-passing programming model. First, communications hardware is inherently used more efficiently in packet mode than in the transmission of single words. Second, programming is far simpler, since the only parallel construct needed is to send or receive a message; this is conceptually similar to the traditional problem of writing to or reading from a working storage medium in programs whose data requirements exceed the core storage capacity. Care must be taken to choose a division that balances the computational load among the processor nodes.

This message-passing model is not, however, without its drawbacks. If the message passing is just a programming construct for the code, it does not matter (as long as a good complier exists). But if actual message passing is done at run-time, there are penalties associated with the additional expensive communications hardware, data copying, and message header information.

Note also that shared memory does not require a static partitioning of memory.

A principal purpose of the directed part of the CHAMMP program is to move various climate models onto various parallel architectures in order to acquire the experience needed to select the most appropriate match of architecture and programming model. In particular, this activity will evaluate parallel implementations of the spectral transform models commonly used for the atmosphere. The Fast Fourier and Gaussian quadrature transforms in such models are global in nature and pose a more difficult communication problem for parallel message-passing hardware than does the kind of local explicit grid-point method described above. In recent years, other problems with spectral transform models have arisen, indicating that it may very well be time to abandon spectral models.

4 PREDICTABILITY

The literature contains a number of versions of what is meant by predictability in climate. Different authors imply different meanings. In the following, we discuss one particular approach to predictability (see Appendix A for a more technical discussion of predictability).

As is well known, the atmosphere is a chaotic, nonlinear dynamical system with a limit of deterministic weather prediction of about ten days to two weeks. Any atmospheric time series of interest for climate prediction must therefore be contaminated by an unpredictable weather noise component. Similarly, any atmospheric GCM run for climate purposes is in fact a weather model for which averages are extracted, and it, too, is contaminated by weather noise. Models that deal directly with climate statistics are free of weather noise, but suffer from a variety of untested closure assumptions.

The problem of weather noise in a GCM is a problem in statistical sampling theory, and most climate sensitivity studies explicitly estimate the statistical significance of the result. This is the simplest aspect of climate predictability, and involves the extraction of a statistically significant signal from a weather noise background that can be reduced only as the square root of the length of the time series or, for models, the amount of computation. The implicit assumption here is that the weather noise is "white" for frequencies much below a few cycles per month. This white noise is associated with the fast physics and dynamics of the atmosphere alone.

Of course, the climate system includes slow components as well, and one of the first questions asked of atmospheric GCMs was whether they responded properly to slowly changing external forcing influences, such as sea surface temperature. This is not a question of theoretical predictability with a perfect model, as discussed above, but rather a question of practical predictability, considering the imperfect nature of the models. Clearly, it is hard to estimate the importance of model imperfections, but an effort is being made through the DOE-supported Program for Climate Model Diagnosis and Intercomparison (PCMDI). Even if, in response to the PCMDI analysis, all models are brought into agreement, there is no guarantee that they do not all err in omitting unknown, but physically important, processes.

The fast response of the atmosphere to changing influences might be estimated from the observed behavior of the atmosphere itself, without recourse to models, by use of the fluctuation-dissipation relation of statistical physics (see Appendix C). The essential idea here is that a complex system will recover from an artificially imposed impulsive disturbance in the same way that it recovers from a naturally occurring fluctuation. In this way, the mean impulse response matrix is estimated as being equal to the observed statistical regression matrix. The atmosphere is not in statistical mechanical equilibrium so that a fluctuation-dissipation relation need not hold exactly, but agreement between the two estimates of the response would considerably improve confidence in model results.

Spatially coherent structures are responsible for observed "persistence" in weather systems (see Appendix B). As far as climate is concerned, those

spatial structures that maintain coherence over periods of years to centuries are particularly important. In contrast to the days or weeks characteristic of spatial structures in the atmosphere, the coherence times for structures in the ocean are much longer. The most clearly recognized slow climate fluctuation in the coupled ocean-atmosphere system is the El Niño/Southern Oscillation (ENSO), and it would be valuable to estimate limits on its predictability. ENSO variability has already been seen to obscure other possible slow global changes.

A natural question then arises about the theoretical limit on the deterministic predictability of the ocean, taking into account the uncertainty in its present state and its time-chaotic behavior. Does the ocean generate irreducible "weather" noise that will obscure other low-frequency signals? The development of global ocean models has just reached the point where the models can be used to estimate this theoretical predictability.

Another rather disturbing possibility is that, as with so many complex systems with many effective degrees of freedom, the climate is in a state of self-organized criticality. In this case, not only is the climate variance possibly divergent at low frequencies, such as $1/f$ noise, but, even worse, could be made up of a series of unpredictable catastrophes of all scales of magnitude. A recently studied prototype of such a system arises as grains of sand are randomly dropped on a sand pile. It is clearly important to look for grains of sand in the climate system; they might cause the present climate research program to grind to a halt.

5 PROGRAM ASSESSMENT

There is clearly great scientific merit in the CHAMMP program, and it is likely that this activity will produce many spinoffs en route to its stated goal. Among these will be the development of a very important infrastructure for applying massively parallel computers to scientific problems. One can expect that libraries of routines, visualization techniques, database methods, and general parallel computing expertise that will be developed during the course of the program will be of importance to the entire scientific community. As noted above, it is also not clear what architectures are best suited to which problems; CHAMMP is directly exploring this issue, at least for the problems relevant to climate (fluid dynamics, radiation transport, database construction, and archiving). The expertise of the DOE labs is particularly well suited to these tasks, and one might expect that they will repeat their performance of the 1960s and 1970s, when they defined and furthered the use of serial supercomputers.

Another important spinoff from CHAMMP will likely be improved capabilities for numerical weather prediction. The codes ported and/or developed by CHAMMP are very similar to (and, in some cases, identical with) those used for operational weather prediction, and the use of massively parallel architectures for that activity will clearly improve present capabilities to handle both synoptic and seasonal scales.

We are less optimistic about CHAMMP's achieving its scientific goal of

an improvement in climate modeling. There are a number of reasons for this skepticism.

First, the spatial and temporal resolution of present or foreseeable GCMs requires numerous parameterizations of sub-grid-scale processes. Some of these are of clear importance (e.g., clouds and their effect on the radiation balance), and their improvement is the focus of intense activity, such as that of ARM. However, it is not clear which other parameterizations are crucial and which are unimportant. There are surely as yet unimagined processes that will have to be incorporated. A reliable climate model must include all of these, and include them correctly. This is a daunting task, far more difficult than simply getting the models to run fast on the latest computer architecture.

Second, the present direction of CHAMMP research (and, indeed, of most climate modeling) is a "brute force" approach in which very high spatial resolution with very short time steps (e.g., 15 minutes) is used to answer questions about behavior on regional scales and decadal time scales. This brute force approach may well not be the best way of understanding the climate system. A useful analogy might be the task of describing the behavior of a gas. The "brute force" approach would involve some potential between the molecules of the gas and integrating the large number of molecular dynamic equations with femto-second time steps. However, if the long term (say, nanosecond) spatially averaged (micron or larger) behavior is of interest, then much simpler and more transparent thermodynamic methods can be applied (see Section 6).

Third, the two most important components of the climate system (the atmosphere and the oceans) have very different dynamical time scales (a few days versus decades). True climate dynamics are therefore likely determined almost exclusively by the ocean, with the atmosphere serving to provide a surface forcing varying stochastically about a mean. Thus a new paradigm beyond simply coupled atmospheric GCMs and ocean GCMs is needed to accommodate the fast physics efficiently.

6 POSSIBLE FUTURE DIRECTIONS OF RESEARCH

In this section, we note areas where CHAMMP could benefit by bringing in scientists who have not been part of the traditional climate modeling community. We believe that the injection of new ideas and concepts is essential if CHAMMP is to near its stated goals.

As we have remarked earlier, most GCMs (and many GCM practitioners) have grown out of traditional numerical weather forecasting. This has led to attempts to understand climate by using computer calculations that repeat a very large number of short time steps of the basic equations of fluid mechanics and radiation transport. There has been very little work attempting to exploit totally different approaches. One could imagine, for example, some sort of analogue of thermodynamics in climate, in which the "brute force" GCM approach is analogous to solving Newton's equations for each molecule in a gas.

No real mechanisms are currently available to encourage and support research directed toward totally new and different ways of studying and predicting climate. We suggest that there is a need for DOE (and/or perhaps NSF) to encourage efforts to break out of the canonical GCM mode by supporting a series of summer schools or workshops on climate. These workshops should be focused as broadly as possible on means other than GCMs to predict the future. Two natural hosts for such a series are the Aspen Center

for Physics and the Institute of Theoretical Physics in Santa Barbara. One should invite a small number of climate phenomenologists to give the physical basis of the problem, but avoid populating these schools with many GCM practitioners. The rest of the attendees would be people largely ignorant of climate research, but drawn from fields that might provide a basis for useful new approaches. These might include physicists interested in many-body physics, statistical physicists, nonlinear dynamicists, and mathematicians with relevant interests (see Appendix D).

The ultimate goal of CHAMMP - to build a new generation of climate simulation codes — requires the best technology and computer hardware, the use of advanced applied mathematics and numerical methods, and improvement in model physics. The present program has little emphasis on applied mathematics and numerical methods. The directed model development program is almost exclusively devoted to porting existing climate codes to massively parallel architectures. In the non-DOE science program, there are only two proposals out of eight (UCLA/CSU and UCSD) that appear to develop genuinely new algorithms. CHAMMP has not involved strong research programs in modeling fluid dynamics, even those at DOE laboratories. There is no lack of interest on the part of these groups in climate modeling. For example, groups at both Los Alamos and Livermore are interested in initiating research programs in numerical algorithm development specifically applicable to ocean and atmospheric circulation problems. CHAMMP should attempt to involve applied mathematicians and numerical analysts in its overall program.

7 LINKS BETWEEN ARM, CHAMMP, AND THE CORE PROGRAM

The Office of Energy Research contributes to the national global change program through the activities of three groups: ARM, concerned with radiation and cloud measurements in support of modeling activities; CHAMMP, centered on improving models through improved software, hardware, and algorithms; and the Core Program, which includes an intercomparison activity among models, as well as support for modeling activities at a number of universities. Both ARM and CHAMMP, which are new programs, have clearly defined management teams that appear to be working effectively as far as the individual programs are concerned. There is no comparable management team for the Core Program, although W. Lawrence Gates has the lead responsibility for the intercomparison program. Robert Cess at Sunnybrook serves as a senior outside advisor to both the intercomparison activities and some aspects of the Core Program.

In order for these programs to be cost effective, it is absolutely essential that there be a close coordination among them. Both ARM and CHAMMP have science teams. One means of achieving coordination is for the two science teams to have common members. Since the science teams are composed of principal investigators (PIs), whether or not overlap is achieved depends on the selection process, which has a stochastic component. It might therefore be advisable to have a few members of one science team designated as members of the other science team, or perhaps to have a few non-PI scientists,

interested in both ARM and CHAMMP, serve on both science teams.

The links of the Core Program to ARM and CHAMMP are less clear. Because of the lack of a well-defined management structure, there does not appear to be a natural way by which the Core Program can interact with ARM and CHAMMP. The linkage is currently best through the model intercomparison program, since it resides at Lawrence Livermore National Laboratory (LLNL), and Michael MacCracken, also of LLNL, is chief scientist of the CHAMMP program. We do not know of any interaction between the Core Program and ARM.

In view of the importance of avoiding duplicate activities, it is essential that the Core Program be closely allied with CHAMMP and, to the extent possible, with ARM. This would require designation of a project manager and chief scientist for the Core Program. These two individuals would then have the responsibility of working with the ARM and CHAMMP teams to ensure close cooperation among the three programs.

A PREDICTABILITY OF CLIMATE

As soon as one asks about the predictability of climate, the question of the meaning of "predictability" arises. At one level, this question is relatively easy to answer, provided that we have some notion or measure of similarity of climates. With such a notion at hand, we can take a climate predictor (such as a GCM), test it experimentally by running it forward a fixed number of years for a large variety of realized initial conditions, see how it performs against true climate using the selected measure of similarity, and so produce a satisfactory numerical measure of its reliability.

While the above concepts are straightforward, they cannot at present be implemented. We do not have a satisfactory notion of similarity of climates (see Appendix B) and no such comprehensive notion is anywhere in sight. The best we can do at present (as is indeed commonly done) is to compare, at a very rough spatial resolution, the temperature or other fields over global or some large regions. Furthermore, the useful database of results is so small relative to the number of variables or possible scenarios as to be ludicrous.

Of course, all physical theories face the same tests that we have just described for climate predictors. Physical theories are just predictors of the behavior of the natural world, and they are tested under a wide variety of circumstances. When they are badly stressed in certain regimes, they often fail and have to be replaced.

Moreover, many physical theories are guided in their construction by

concern for Ockham's razor — the assumptions going into them are relatively few and mostly visible. When they fail, it is often possible to see which assumptions were at fault. (Invisible or unperceived assumptions cause more difficulty.)

Nothing like this holds for climate models. The number of assumptions, tuned parameters, approximations, etc., defy description. The detailed operation of the model is never seen — only the computer "knows" what is going on. And given how difficult it is to make these models function satisfactorily in normal circumstances, what grounds are there for believing their performance in stressed regimes (such as doubled CO_2)?

Yet what can we do about predicting climate? One can look for some sharper insights than those models give us — while they are an audacious attack on the problem, they are still mostly brute force — and hope for more inspired predictors based on perceiving general principles. But until these come along we are stuck with GCMs. (We do not wish to suggest, however, that since GCMs are the only tool available just now, they are the right tool for now.)

In traditional statistical theory, when a predictor for a random variable is given, it is analyzed by finding the distribution function of the error, or at the very least, the variance of the error. Nothing like this has ever been done in GCMs. Even though they are not totally deterministic, we are presented with a run and told, "This is the climate prediction." It is, of course, very expensive and time-consuming to run GCMs forward many years, but so is making incorrect policy decisions based on their predictions. At the very

least, then, models should be run with a large variety of initial conditions, slightly offset tuning parameters, etc., in order to see the change (or study the variance) in their predictions. And when clouds are put into models more realistically, a little stochasticity in cloud distribution can be investigated as well. We will have to improvise some measure of distance between climates to quantize the variance.

We have been discussing one aspect of predictability, i.e., assessing the virtue of a given predictor. (We will subsequently give an illustration with a measure of similarity for general dynamical systems.) Though even this task is somewhere between difficult and impossible for GCMs, it does not come close to addressing the real problem of predictability — is there an optimal predictor, and what is its behavior and reliability?

Finally, we do not see any way of deciding for a general dynamical system whether there is an optimal predictor. Fortunately, however, there is an optimal predictor for general stationary stochastic processes in the category of linear predictors (for fully Gaussian processes it is optimal). It is measured by the limiting variance of the Wiener predictor of a state as more and more of the past is known. There is a relatively simple theory described in great detail in texts [1] on stochastic processes, which tells us that the limiting variance of the Wiener least squares predictor is:

$$e^{\int_{-1/2}^{1/2} \ln P(\alpha) d(\alpha)}$$

where we are given a stationary, discrete, real-valued stochastic process with auto covariance

$$R(n) = E[(x(p)x(p+n))]$$

and spectral density

$$P(\alpha) = \sum_{n=-\infty}^{\infty} R(n) e^{2\pi i n \alpha} .$$

Notice that by Jensen's inequality

$$\int_{-1/2}^{1/2} \ln P(\alpha) d(\alpha) \leq \ln \int_{-1/2}^{1/2} P(\alpha) d(\alpha) = \ln R(0) ,$$

as the limiting Wiener predictor is strictly less than the variance of the individual random variables for all processes except the uncorrelated noise process. Notice that if there is a set of positive measurements in which $P(\alpha)$ is zero, then perfect prediction is possible. Thus if the power spectrum is not broadband, we are in good shape.

Unfortunately $\int_{-1/2}^{1/2} \log P(\alpha) d(\alpha)$ is very sensitive to estimation of $P(\alpha)$ when the latter is small, so the spectral density has to be compiled carefully. It might be advisable to allow for error in measurement amounting to a superimposed uncorrelated noise process of power σ^2, in which case the power spectrum should be replaced by $P(\alpha) + \sigma^2$. Choice of σ^2 depends on circumstances.

Just for illustration, we computed this measure of predictability for the Lorenz-27 model, using a 25,521-year temperature record it produced, and for the climate using a 134-year temperature record, the latter being quite shaky for accurate estimation of $P(\alpha)$. The results were in the first case a variance of e^{-9000}, and in the second e^{-100}.

The conclusion for the Lorenz is not after all too surprising. It is a deterministic process, and an infinite record should be enough to determine the dynamics precisely.

There are also formulas for variance of prediction at lags greater than one, but they are somewhat more complicated and we refer the curious reader to the literature [1]. Computing the variance of the Wiener predictor for a fixed finite past is an easy exercise in linear algebra.

It is worth noting that the optimal linear predictors we have described above are not in all cases optimal predictors. It is known that there are (non-Gaussian) stationary stochastic processes that are not perfectly predictable linearly, but for which a perfect nonlinear predictor exists.

Next we want to describe briefly predictability for dynamical systems, solely in terms of measurement error. Here, not unexpectedly, the Kolmogorov entropy and Lyapounov exponents play important roles, yet in many ways the conclusions are misleading — a point we will discuss more carefully later.

So let us suppose we have a vector-valued dynamical system x_t, with dynamics completely known. The vector variable is observed at time t to have value a_t. How do we predict the value $x_{t+\tau}$, on the assumption that all our observations are subject to a uniform observation error ϵ? What we are saying is that x_t is a random variable uniformly distributed in a ball of radius ϵ centered at a_t. Now, of course, the natural predictor for $x_{t+\tau}$ is $F_\tau(a_t)$, where F_τ is the deterministic evolution of the system at interval τ. But we suppose we have the same uniform measurement error of radius ϵ centered at $F_\tau(a_t)$. With these conventions, we can give *precisely* the probability that our predictor is correct. It is just the relative measure in the ball of radius ϵ centered at a_t of those points that land τ time units later in the ball of

radius ϵ centered at $F_\tau(a_t)$.

In order to assess the merit of this predictor, we should average in some sense (probably with respect to an invariant measure) over the position a_t of the initial point, but on the assumption that the system is ergodic, we shall see that such averaging is not required, if we take the limits $\epsilon \to 0$ and $\tau \to \infty$. While these kinds of asymptotics are customary in the study of dynamical systems, it is important to point out that they are unrealistic for the problems we really face. But the problem of computing predictor reliability for reasonable ϵ and τ requires massive computation of orbits and choice of initial points. These are precisely the kinds of computations that are *not* made for climate models, but that are sorely needed.

Let us go back to the computation we need. We want to compute:

$$\int_{\substack{|x-a|<\epsilon \\ |F_\tau(x)-F_\tau(a)|<a}} dx_1 dx_2 \ldots dx_n / \int_{|x-a|<\epsilon} dx_1 dx_2 \ldots dx_n.$$

Now to a first approximation $F_\tau(x) = F_\tau(a) + L_\tau(x-a)$, with L_τ being the Jacobian of $F_\tau(x)$ at $x = a$. So to a first approximation the numerator is

$$\int_{\substack{|x|<\epsilon \\ |Lx|<\epsilon}} d\vec{x}.$$

The approximations are good in the limit $\epsilon \to 0$. The integral above is somewhat unpleasant to evaluate exactly, so we will approximate it by another volume. Suppose L^*L has eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. The volume we want is the intersection of the ellipsoids:

$$x_1^2 + x_1^2 + \ldots + x_n^2 \leq \epsilon^2$$

$$\lambda_1 x_1^2 + \lambda_2 x_2^2 + \ldots \lambda_n x_n^2 \leq \epsilon^2,$$

which we will replace by volume of corresponding rectangular parallelepipeds:

$$|x_1| < \epsilon, \ldots, |x_n| < \epsilon$$

$$|\sqrt{\lambda_1}x_1| < \epsilon, \ldots, |\sqrt{\lambda_n}x_n| < \epsilon.$$

This is easily computed as

$$\frac{\epsilon^n}{\prod_{\lambda_i>1} \lambda_i^{1/2}},$$

and, upon division by the adjusted volume of the ball, gives the probability in question as

$$[\prod_{\lambda_i>1} \lambda_i]^{-1/2}.$$

Now for large τ, the $\lambda_i^{\frac{1}{2\tau}}$ are the exponentials of the Lyapounov exponents and, by Pesin's Theorem, the sum of the positive Lyapounov exponents is the Kolmogorov entropy H, so that approximately, for small ϵ and large τ, we have the probability of correct prediction is $e^{-\tau H}$. This vanishes very rapidly with increasing τ, if there are positive Lyapounov exponents. (The same or a similar result can be obtained from a variety of other points of view, such as rate of decay of information.)

As we noted above, the asymptotics $\epsilon \to 0$, and $\tau \to \infty$ are unrealistic in practice. There is an inescapable error of measurement in climatology (molecular rather than atomic) and predictability is really only of interest at relatively short time scales, say 20–50 years.

The asymptotic statements of the well-developed parts of a theory of dynamical systems are not really pertinent to the problem of climate prediction. They have given us a language for descriptive purposes, and an inkling

of the complexity of behavior possible, but long-term dynamics is still largely a mystery.

There is, however, one sense in which a general theory of dynamical systems may be of genuine assistance. We are generally comfortable making predictions in dynamical systems having no positive Lyapounov exponents, and when we form long-term running averages, in a sense we are trying to depress the exponents. The question this naturally raises is this: how does one suppress (or depress) the large Lyapounov exponents? More precisely, given a dynamical system x_t, are there interesting functions H of k variables so that the derived system

$$y_n = H(x_n, x_{n+1}, \ldots, x_{n+k-1})$$

has only small exponents?

B SPATIAL STATISTICS

Spatial structures spontaneously arise in spatially extended nonlinear dynamical systems as a consequence of the nonlinearity of the equations of motion. These structures can be quite robust against perturbations and survive for long periods of time, even when no comparably long time scales appear to be present in the equations of motion (persistence). Robust spatial structures can persist even though the fluid flow is turbulent. To a large extent, these spatial structures determine the long-time behavior of the system.

The persistent spatial features in atmospheric flows are certainly those we see in weather maps: high and low pressure systems, warm and cold fronts, etc. Highs and lows do not survive for long periods of time, but they survive long enough to provide the phenomenon of persistence (see JASON Report JSR 91-340). The most important long-lived nonlinear spatial structures in the atmosphere are probably the systems of large-scale convective cells, which transport heat away from the tropics toward the North and South Poles. Our knowledge of spatial structure in the ocean is less than that in the atmosphere, due to the difficulty of obtaining data. The primary example of a long-lived structure in the ocean may be the so-called "Conveyor Belt" that flows deep down the center of the Atlantic, around Antarctica, into the Pacific Ocean, under India, and up along Africa and Europe. Other examples include various persistent currents, such as the Gulf Stream. All of these spatial structures have a long-term effect on climate. The principal

GCMs should be able to tell us how spatial structures change with changing atmospheric composition.

The principal products, and in some cases the only products, of GCMs are maps representing two-dimensional fields over the surface of the earth. Calculated fields are compared with observed fields with the usual incantations: "The maps look similar, or "They do not look at all alike." People have been drawing maps and studying spatial parallels for millennia. Computers have drawn maps for decades. Far less attention has been paid to analyzing these spatial patterns in terms of statistics than has been paid to the analysis of time series. The human eye and brain form a wonderful mechanism to analyze and recognize patterns. However, they are subjective and likely to err, perhaps in the wished-for direction.

Given the importance of intercomparison of two-dimensional data fields, it is essential to develop ways of translating these fields into a few numbers that provide a clear and useful summary of the information contained within these fields. The development of such statistics would be essential in a rigorous analysis of how well GCMs compare with each other and with reality.

Spatial statistics are likely to differ from time series statistics in a number of important ways. In time series statistics, the data very often show only short-range dependence; the auto correlation drops off very sharply. If one examines spatial patterns such as meteorological pressure fields or topographical elevation, it is quite clear that the spatial fields show long-range dependence. This implies that many of the approximations used in time series analysis that require the absence of long-range dependence are not

applicable to spatial statistics.

Continuity is a concept of great importance in time series analysis. When one goes from continuous to finite sample data, the requirements for stationarity carry over. With spatial data, a sampling scheme may alter stationarity and isotropy. Very great care is thus required to translate results obtained from continuous variables to results applied to variables sampled over finite spatial integrals.

In time series analysis, it is very clear how one carries out the transition to obtain asymptotic results. This is not the case in spatial problems. Consider the finitely sampled spatial region with $m \times n$ sample points. For asymptotic results, clearly either m or n should tend to infinity, but very little can be concluded about the ratio m/n. This feature appears in a sensitive way in such questions as determining the asymptotic mean.

With the advent of computers, a great deal has been done in the area of pattern recognition. At the same time, very little progress has been made in answering such questions as what kinds of statistics should be used to compare the "likeness" of different spatial patterns. Because spatial patterns are a primary result of GCMs, we believe that a research effort should be mounted to pursue questions related to spatial statistics.

C A FLUCTUATION-CORRELATION RELATIONSHIP

C.1 Introduction

It is often useful to have a way to estimate the way a nonlinear system, such as the climate or weather systems, will respond to variations in the forcing function. With something like the dynamics of climate, we are interested, for example, in the way a dynamical variable, such as the global average temperature, will change if some quantity such as the solar constant or the CO_2 concentration or whatever is slightly varied. In this section we derive a relationship between some correlation functions of dynamical variables of a nonlinear system and the variations of that dynamical variable with respect to a fluctuating force. No assumptions about the statistical state of the nonlinear system are required, though we do assume that the force fluctuates as a Gaussian, white process.

The traditional *fluctuation-dissipation* theorem relates correlation functions to rates of relaxation to an "equilibrium" state. To see that relationship in action, we consider the simplest Langevin equation for a linear process:

$$\dot{q(t)} = -\alpha q(t) + \eta(t), \qquad (C-1)$$

where $\alpha > 0$ is a dissipation rate, $q(t)$ is the dynamical variable, and $\eta(t)$ is

Gaussian, white noise with zero mean and

$$< \eta(t)\,\eta(t') >= \sigma^2 \delta(t - t'). \qquad (C-2)$$

The solution to the differential equation is easily seen to be

$$q(t) = \exp[-\alpha t] q_0 + \int_0^t dt' \exp[-\alpha(t' - t)]\eta(t'). \qquad (C-3)$$

This leads to

$$< \dot{q}(t)\dot{q}(t) >= \frac{\sigma^2 \alpha}{2}, \qquad (C-4)$$

when transients have died away. If we identify the average of the velocity squared with $\frac{k_B T}{m}$ (k_B is Boltzmann's constant and T is the temperature of the "bath" in which the system is immersed), then we have $\sigma^2 = \frac{2k_B T}{m\alpha}$, which relates the fluctuations σ^2 to the dissipation α. There is a clear assumption of the dynamical degrees of freedom, here just $q(t)$, having come into "equilibrium" with a heat bath of temperature T. Basically this means that the fast degrees of freedom, represented by $\eta(t)$, are thermally driving the slow degrees of freedom ($q(t)$).

C.2 A Fluctuation-Correlation Relationship

Here we start with a general one-dimensional nonlinear equation of motion. The result holds for d-dimensional equations as well with a shift in notation and the introduction of some correlation tensors. Skipping these tensor niceties, we assume that the dynamics of $q(t)$ satisfies

$$\frac{dq(t)}{dt} = F(q(t)) + \eta(t), \qquad (C-5)$$

and that, once again, $\eta(t)$ is Gaussian, white noise with a standard deviation σ^2.

We introduce the probability density in q space $p(qt|q's)$, which for $s > t$ is the probability that at time s the system is at q' if it was at q at time t. This is a kind of "propagator" for the system.

A functional integral representation for this propagator is given by the following reasoning [see B. Jouvet and R. Phythian, *Phys. Rev.* **A19**, 1350 (1979)]. Any function of $q, g(q)$ can be represented by

$$g(q) = \int D[q']g(q')\delta(q - q'), \qquad (C-6)$$

where this is a functional or path integral. This involves the solution $q(t)$ along the paths in the integral, so we switch to an integral that uses the equations of motion as a constraint to keep the orbits in $q(t)$ space along allowed paths. As argued in the paper cited, we have

$$g(q) = \int_{q(0)=q_0} D[q']\, D[p]g(q') \exp[i \int_0^T dt' L(t')], \qquad (C-7)$$

with an effective Lagrangian

$$L(t) = p(t)(\dot{q(t)} - F(q(t)) - \eta(t)), \qquad (C-8)$$

and the "momentum" is a Lagrange multiplier that enforces the equations of motion. The advantage of this formulation is that the variation of any function of $q, g(q)$, with respect to the fluctuating quantity is seen to be an average over a factor $p(t)$ in the functional integral (up to a factor of $-i$). Thus we have

$$\frac{\delta q(t_1)}{\delta \eta(t_2)} \propto \int D[q']D[p]q'(t_1)p(t_2)\exp[i\int_0^T dt' L(t')]. \qquad (C-9)$$

Next we introduce a "Schrödinger" representation in which factors of p become just $\frac{\partial}{\partial q}$ and

$$p(q_1t_1|q_2t_2) \propto \int_{q(t_1)=q_1;q(t_2)=q_2} D[q]D[p]\exp[i\int_{t_1}^{t_2} L(t)dt]. \qquad (C-10)$$

Now one can carry out the averages over the fluctuations of $\eta(t)$ and arrive at

$$\langle\frac{\delta q(t_2)}{\delta \eta(t_1)}\rangle = -\int dq_1 \int dq_2 q_1 p(q_1t_1|q_2t_2)\frac{\partial p(q_2t_2|q_0t_0)}{\partial q_2}. \qquad (C-11)$$

Similarly

$$\langle q(t_1)q(t_2)\rangle = \int dq_1 \int dq_2 q_1 p(q_1t_1|q_2t_2)q_2 p(q_2t_2|q_0t_0). \qquad (C-12)$$

Along with the Schrödinger representation is a "Schrödinger equation," namely the Fokker-Planck equation for these processes. We have the "forward" equation

$$\frac{\partial p(q_1t_1|q_2t_2)}{\partial t_2} + \frac{\partial(F(q_2)p(q_1t_1|q_2t_2))}{\partial q_2} = \frac{\sigma^2}{2}\frac{\partial^2 p(q_1t_1|q_2t_2)}{\partial q_2^2}, \qquad (C-13)$$

and the "backward" equation

$$\frac{\partial p(q_1 t_1|q_2 t_2)}{\partial t_1} + F(q_1)\frac{\partial p(q_1 t_1|q_2 t_2)}{\partial q_1)} = -\frac{\sigma^2}{2}\frac{\partial^2 p(q_1 t_1|q_2 t_2)}{\partial q_1^2}. \qquad (C-14)$$

Using these two differential equations for the probability densities in the expression for

$$\langle q(t_1)q(\dot{t}_2)\rangle = \int dq_1 \int dq_2 q_1 q_2 [\frac{\partial p(1|2)}{\partial t_2} p(2|0) + p(1|2)\frac{\partial p(2|0)}{\partial t_2}], \qquad (C-15)$$

we arrive after a few integrations by parts at

$$\langle q(t_1)q(\dot{t}_2)\rangle = \langle q(t_1)F(q(t_2))\rangle + \sigma^2\langle\frac{\delta q(t_1)}{\delta\eta(t_2)}\rangle, \qquad (C-16)$$

which is our "fluctuation-correlation" relation. It relates properties of the equation of motion in the presence of the fluctuating forcing $\eta(t)$ — namely, in our context, climate evolution in the presence of fluctuating forcing due to weather — to variations of the dynamical quantities of climate, here represented by $q(t)$, with respect to changes in that forcing. This relation could, of course, be tested by explicit GCM calculations and, if successful could be used to more efficiently estimate the sensitivity of results to change in the external forcings.

D CONFIGURATION SPACE VOLUMES AND THE STATISTICS OF EXTREMES

Geodesic flow on a Riemannian manifold is a classical example of a deterministic dynamic system. A point on the manifold, with local coordinate $\vec{q}$, determines a configuration. A (co)-tangent vector $\vec{p}$ specifies the dynamic or state and the geodesic line along (dual to) $\vec{p}$ describes the time evolution of the state. The case that the manifold M^n is noncompact but still has finite Riemannian volume is particularly interesting, since we may regard excursions far away from some base point $* \in M$ as representing extremes whose statistical behavior can be studied using the finite invariant Riemannian measure on the unit tangent bundle T_1M. Locally this measure is proportional to $dvol_M \times dvol_{S^{n-1}}$ where the second factor is the homogeneous volume element on the $(n-1)$-dimensional sphere. It is a classical theorem of Liouville's that geodesic flow $\Phi_t : T_1M \to T_1M$ preserves this measure.

Interesting examples of such M are obtained as $SO(2) \setminus PSL(2,C) / \Gamma$ where Γ is a con-finite volume (but not co-compact) and torsion free. (The assumption that Γ contains no torsion makes the quotient a manifold, rather than an orbifold. Except for notation, our discussion would be nearly identical in the orbifold setting so the reader may drop the additional hypothesis. If this is done the best known example is $\Gamma = PSL(2,Z)$. If one prefers to work with genuine surfaces, Γ contains many torsion free subgroups of finite index $\Gamma' \subset \Gamma$.)

We may formulate the following elementary theorem on the excursions of geodesic flow on M.

Theorem 1. Let $f(x)$ = volume $(M \backslash Ball_x(*))$ and let $h(t)$ be any function (called "excursion limiter") $h : Z^+ \to R^+$ such that $\sum_{t=1}^{\infty} f(h(t))$ is finite. For almost every $*' \in M$, the set of tangent directions v such that $dist(*, \phi_i^v(*')) \geq h(t)$ for infinitely many i has Lesbeque measure zero. ($\phi_i^v(y)$ = proj $(T_1 M \to M) o \Phi_t(y, v)$; that is, the point y flows to in direction v after time t.)

Proof: Let $S_i, i = 1, 2, \cdots$, be the subset of $T_i M$ that will flow after time i outside of $B_{h(i)}(*), S_i = \Phi_i^{-1}(M \backslash B_{h(i)}(*))$. Since Φ_i is volume preserving, $vol(S_i) = f(h(i))$. Let χ^j be the characteristic functions for S_j and $\chi_N = \sum_{j=1}^{N} \chi^j$.

By hypothesis

$$\sum_{t=1}^{\infty} \int \chi^t = lim_N \int \chi_N - \sum_{t=1}^{\infty} \int \chi^t = finite.$$

By the monotone convergence theorem:

$$lim_N \int \chi_N = \int lim_N \chi_N$$

It follows that $lim_N \chi_N$ exists (and is finite) except on a set E of measure zero in $T_1 M$. But $(y, v) \in E$ iff geodesic flow from y in direction v satisfies the theorem's inequality infinitely often. By Fubini's theorem almost all directions v at almost all points $*'$ satisfy $(*', v) \in E$.

We return to our examples: finite area hyperbolic surfaces with cusps $(SO(2) \quad PSL(2, C) \ / \ \Gamma)$. In this case it is known from Margulis's analysis

of cusps [2] that f is proportional up to universal constants to e^{-x}. For any $\epsilon > 0$ we find an excursion limiter $h(t) = (1+\epsilon)\ log\ t$. (Since $\sum_{\infty}^{t=1} e^{-(t+\epsilon)log\ t} = \sum_{\infty}^{t=1} \frac{1}{t^{1+\epsilon}}$ converges.) Thus it is vanishingly rare to find a geodesic ray on a finite volume hyperbolic surface that heads toward infinity appreciably faster than $log\ t$.

One may ask whether or not we should expect excursions to occur at the maximal rate consistent with the preceding volume considerations (theorem 1). The answer is "yes" if the mixing properties of the flow are sufficient to make individual penetrations of the ends essentially independent in the sense of the Borel-Cantelli theorem [3]. In searching the literature, we discovered that Sullivan [4] has obtained this type of result for finite volume hyperbolic manifolds M^d of any dimension $= d$. Let us write $v(t)$ for the image in M of geodesic flow along v for time t.

Theorem (Sullivan). For geodesic flow on a finite volume hyperbolic manifold M^d we have:

$$\lim_{t\to\infty} \sup \frac{\mathrm{dist}(*, \mathrm{v(t)})}{\log \mathrm{t}} = 1/(\mathrm{d}-1).$$

for almost all $v \in T_1 M$.

We give — in outline — an argument near Sullivan's. (We have attempted to match his notation where possible and have restricted to dimension $d = 2$ for convenience.)

Lemma 1 (Borel-Cantelli). Let A_i be subsets of a measure space $i = 1, 2, 3....$ Let $A_\infty = \{x | x \in A_i \text{ for infinitely many } i\}$. Suppose there is a $c \geq 1$

so that for $i \neq j |A_i \cap A_j| \leq |A_i|\,|A_j|$. Then $\sum_i |A_i| = \infty$ iff $|A_\infty| > 0$.

Proof: The "only if" direction is the nontrivial one.

As before, let χ_N be the sum of characteristic functions on $A_i, i \leq N$.

$$\int \chi_N^2 \leq \sum_{i \leq N} |A_i| + c \sum_{i<j \leq N} |A_i||A_j| \leq c\left(\int \chi_N\right)^2$$

so $\| \chi_N \|_2 \leq \sqrt{c}\, \| \chi_N \|_1$.

Set $\psi_N = \chi_N / \| \chi_N \|_1$ and let ψ be a weak limit in the ball of radius $\sqrt{c}$ in L^2. Since $\langle \psi_n, 1 \rangle \to \langle \psi, 1 \rangle$ we have $|\psi|_1 = 1$. The limit $\lim_N |\chi_N|_1 = \infty$ by hypothesis so χ_N must approach ∞ on support $\psi = A$. Thus A is a set of positive measures that certainly lies inside A_∞.

We now need to exploit hyperbolic geometry. Margulis used the Anosov structure of the geodesic flow on $\mathbf{H}^d$ to prove various ergodicity and mixing assertions. His investigation gives the following information on the (Euclidean) distribution of horoballs in the Poincaré disk model of hyperbolic space.

Lemma 2. Let M be a hyperbolic surface of finite volume. The cusps have neighborhoods that lift to a disjoint union of horoballs (Bs) in the unit disk D^2. These disks, B, are disjoint, tangent to ∂D^2, are copious in all size scales. The last assertion says there is a universal constant $O < \rho < 1$ so that there are approximately (up to further universal multiplicative constants) ρ^{-n} horoballs of diameter in the range (ρ^n, ρ^{n-1}).

A geodesic on M lifts to a boundary-orthogonal semi-circle C in D. As C

comes in toward the ideal boundary, we may assume locally (only a very small error is introduced here) that we are seeing a ray coming perpendicularly into the x-axis in the upper half-space model.

In the half-space model, we note that penetration toward a cusp is comparable*[1] to $\frac{\log d}{d'}$ as indicated in Figure 1.

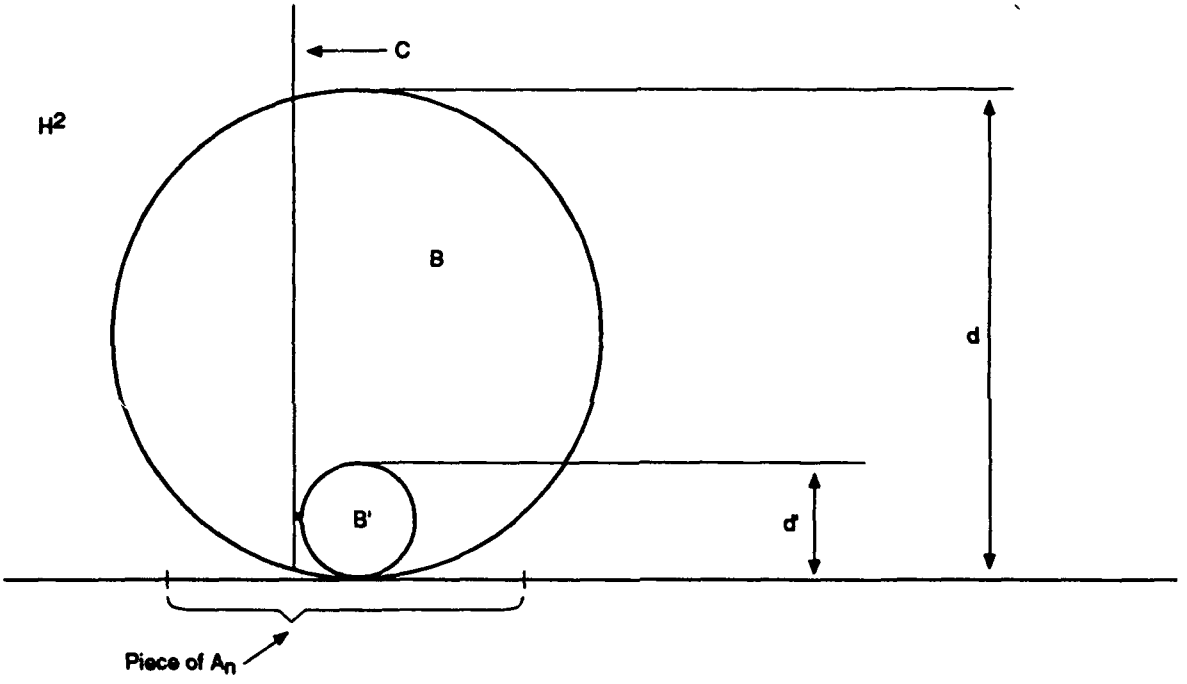

Figure 1.

As in Lemma 2, parse the Bs according to the scale of the diameter $d \in (\rho^n, \rho^{n-1})$. Let $S_n, n \geq 1$, be the union of all Bs of scale n (as above). Let A_n be the disjoint union of intervals in [0,1] of diameter $-\frac{\rho^n}{\log \rho^n}$ centered at the point of contact between S_n and the real line.

[1] *Means the ratio is bounded above and below by universal constants.

According to Lemma 2 $|A_n|$ is comparable to $\frac{1}{-log\ \rho^n}$ so $\sum_{i=1}^{\infty}|A_n| \sim \frac{-1}{log\ \rho}\sum_{i=1}^{\infty}\frac{1}{n} = \infty$. Furthermore the "independence" hypothesis of Lemma 1 will be verified in the next paragraph.

Suppose $m > n$. Ignoring fixed constants, how many intervals of A_n can meet an interval of A_m? Because the smaller intervals have ρ^m spacing and the larger have size $= \frac{-\rho^n}{log\ \rho^n}$, max $\left(1, \frac{-1}{log\ \rho^n}\frac{\rho^n}{\rho^m}\right)$ is an upper bound. In fact, we claim that if the two intervals meet, then (up to universal multiplicative constants) $\frac{-1}{log\ \rho}\frac{\rho^n}{\rho^m} > 1$.

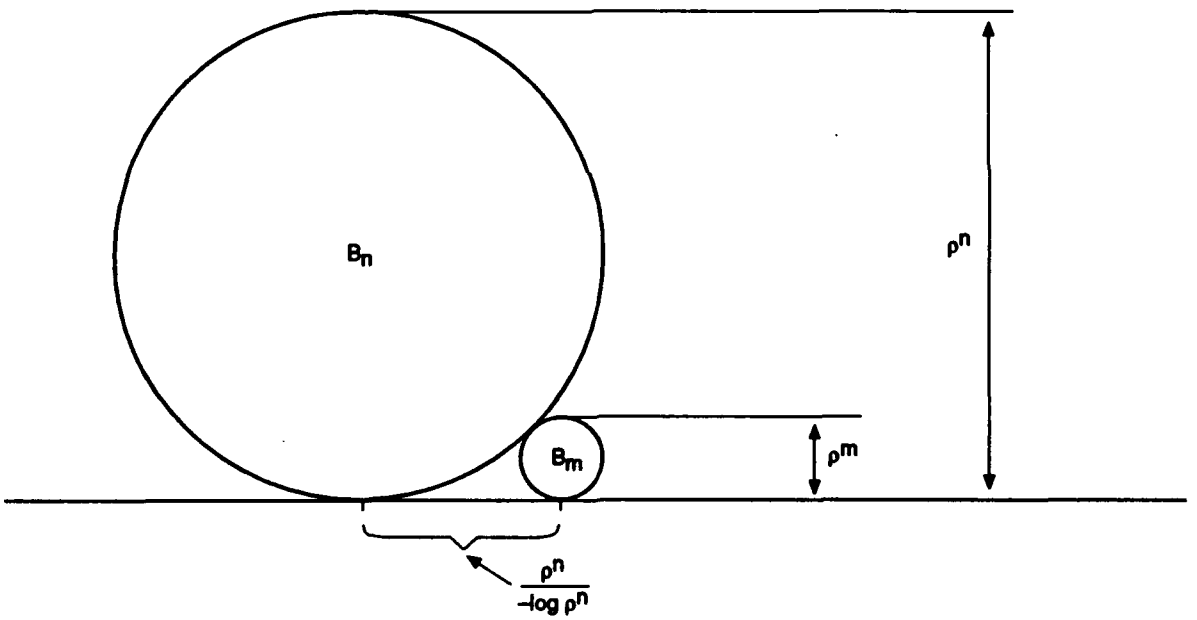

Figure 2. Geometry of A_m meeting A_n.

From the geometry above, Figure 2,

$$\rho^m \approx \left(\frac{\rho^n}{-log\ \rho^n}\right)^2$$

so

$$\frac{\rho^n}{-log\ \rho^n}\left(\frac{-1}{log\ \rho^n}\ \frac{\rho^n}{\rho^m}\right) \equiv 1$$

therefore

$$\frac{\rho^n}{\rho^m}\left(\frac{-1}{log\ \rho^{2n}}\right) > 1.$$

With the above maximum replaced (up to a fixed multiplicative constant) by the second term alone, we have an upper bound for $|A_n \cap A_m|$ based linearly on the length of the larger intervals and the spacing of the smaller. This yields the required "independence":

$$|A_m \cap A_n| < C|A_m|\ |A_n|.$$

It now follows from Lemma 1 that A_∞ has positive measure. Referring back to the Poincaré model, we have the analogous sets $\tilde{A}_i \subset \partial D$. It does not seem to be possible to make these sets entirely natural (i.e., Γ invariant) since distance into a cusp is only definable up to constants, but one can, by replacing $log\ p^n$ by $(1+\epsilon)log\ \rho^n$, produce large and small versions, $\partial D \supset \tilde{A}_i^-$, with $\gamma(\tilde{A}^- i) \subset \tilde{A}^+ i$ for all $\gamma \in \Gamma$. Now define $\tilde{A}_\infty^\pm = \{x \in \partial D | x \in \tilde{A}_i^\pm$ for infinitely many $i \geq 1\}$. Again $\gamma\tilde{A}_\infty^-$ for all $\gamma \in \Gamma$. And as above $|\tilde{A}_\infty^-| > 0$. This together with the topological transitivity of Γ on ∂D implies $|\tilde{A}_\infty^+| = 1$.

Thus almost all trajectories penetrate a (B, B') pair (see Figure 1), where B has size k (i.e., diameter $\in (\rho^k, \rho^{k-1})$ and B' is smaller by a factor of $(\frac{1}{-log\ \rho^k})$ for infinitely many k. The time it takes for the trajectory to reach B' is up to an additive constant: $t = -log(\frac{\rho^k}{-log\ \rho^k})$. The cusp penetration achieved with this excursion is (up to an additive constant): $p = -log(\frac{1}{-log\ \rho^k})$. That is $t = -k\ log\ \rho + log(-k\ log\ \rho)$ and

$p = log(-k\ log\ \rho) = log\ t - log\ log(-k\ log\ \rho)$ for infinitely many ks. This estimate completes the proof of the theorem.

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